A Mirror for Illuminating Opaque Objects for the Projecting Microscope.

By Persifor Frazer, Jr.

(*Read before the American Philosophical Society, Feb.* 20, 1880.)

The subject of the present note is an arrangement for representing opaque objects through the gas microscope, especially adapted to Zentmayer's $1\frac{1}{2}$ inch objective. It is only claimed to be better than the parabolic reflector of Smith & Beck, J. Lawrence Smith, Sorby and others, where the working distance of the microscope is comparatively large (*i. e.*, the distance from the objective to the object on the stage is $\frac{1}{2}$ inch or more) and for the purposes mentioned. Where the distance is as great as that just mentioned the dispersion of rays from the reflexion at one point, of rays from very different parts of the mirror, is so great that only a few rays from the upper part of the mirror reach the lens at all. It would be different with a lens having a very small working distance, and in this case a parabolic reflector would be preferable.

The apparatus consists of a brass tube made to slide over the lens, on the lower end of which is fixed a glass plate about 1 mm. in thickness so attached as to be capable of a sliding motion towards or away from the hinged mirror which is attached to the edge of the metal flange in which the glass plate slides. This simple contrivance permits the glass plate to be brought into close contact with the reflecting mirror no matter at what angle the latter may be placed.

The mirror is made of nickel-plated German silver neatly mounted on a small hinge.

The light is admitted from below through a diaphragm after the rays have been rendered parallel by the condenser of the lantern, the aperture of the diaphragm being adapted to the maximum thickness of beam which can be effective for illumination, and which (calling a the aperture of the lens and i the angle of incidence of the beam) $= a \cos i$: or for an aperture of $\frac{7}{8}''$ $(= 0.875'')$ and an incident angle of 62°, 0.411″ or roughly 0.4″.

The less the incident angle of course the larger the beam of light will be, and the greater diameter of the diaphragm. The refractive index of the glass employed to make the plate being 1.5, in order that the critical angle 41° 48′ may not be exceeded in the refracted ray, this angle of incidence or i must not be less than 61° 51′ or roughly 62°.

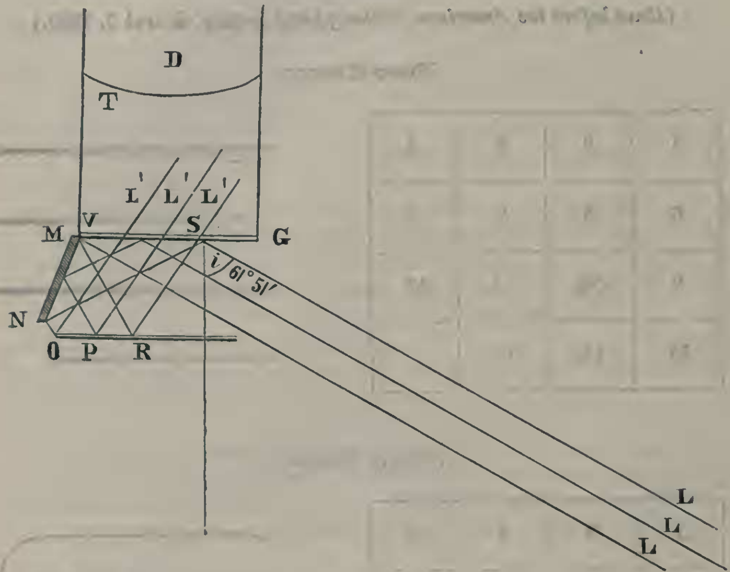
This minimum value of i determines the area of surface which can be illuminated on the microscope stage, but by altering the angle of the mirror very slightly all parts of the object may be successively projected on the screen. This minimum value is easily obtained from the critical angle of the glass employed, which is 41° 48′. The complement of this, or 48° 12′, is equal to the angle of refraction (or r) when the minimum value of i is attained.

$$\frac{\sin\ i}{\sin\ r} = 1.5$$

$$\sin\ i = 1.5\ (\sin\ 48° \ 12')$$

$$i = 61° \ 51'.$$

In other words, the angle between the luminous ray and the glass plate can never exceed 28° 09′, or in round numbers 28°.



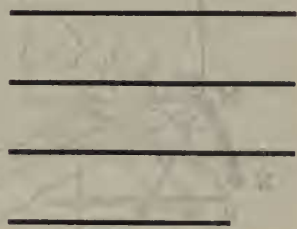
GM. Cover glass.
MN. Reflecting mirror.
OPR. Reflexion on object.
L′L′. Rays which pass through the objective.
D. Lens.
T. Sliding tube carrying reflecting mirror.
Angle of incidence 62°.

Three Methods and Forty-Eight Solutions of the Fifteen Problem. By Persifor Frazer, Jr.

(*Read before the American Philosophical Society, March* 5, 1880.)

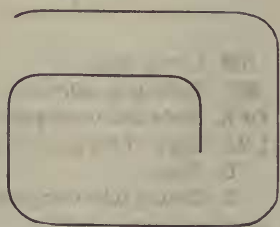
First Method.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	



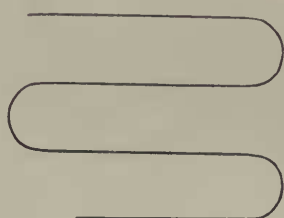
Second Method.

1	2	3	4
12	13	14	5
11		15	6
10	9	8	7



Third Method.

1	2	3	4
8	7	6	5
9	10	11	12
	15	14	13



The requirements of the popular Fifteen Puzzle are to "move the blocks until in regular order." This regular order may be of three kinds, when the numbers are in consecutive series and the blank space is left either at the beginning or the end :

1. That usually understood where the numbers follow each other in broken lines like reading matter in type, or in the opposite direction.

2. Where the numbers follow a single coil from some point on the edge of the box to the centre, or *vice versa.*

3. Where the numbers follow a zigzag course across the box, reading from left to right on the first and third lines, and from right to left on the second and fourth, or *vice versa.*

It can be shown that the three conditions which render possible one or the other of these solutions are : 1st, the number which heads the outside column, whether 1 or 15 ; 2d, the direction in which the numbers increase along the outside, whether with or against the motion of the hands of a watch ; 3d, the order in which the four middle numbers occur.

Note. The direction of the motion of the column itself must always be such that the head does not move to a square just vacated by one of the series of which it is the first or last member.

There are sixteen possible solutions under each of the three methods, eight of them applicable to cases where 1 is at the head, and eight to cases where 15 is at the head.

Of these eight solutions four only are possible in any given position of the middle blocks in the box, the other four becoming possible when the positions of any two of the middle blocks are exchanged.

Of each group of four possible solutions in any given position of the middle blocks, two are possible when the outside column of numbers increase in magnitude with the motion of the hands of a watch, and two when the increase is the reverse of this.

Finally of the two possible solutions where the outside numbers are in arithmetical series and the position of the middle blocks is given, one brings the head of the outside column to a given corner, and the other to the diametrically opposite corner.

The middle numbers must always be the lowest four or the highest four.

The following four tables comprise all possible groupings of these two sets of four numbers. The dots in the small squares indicate by their number the method by which the solution can be obtained without disturbing the middle blocks, provided the outside numbers have been put in the proper order :

TABLE I.

	1.	2.	3.
a	12 · 15 14 13	12 ··· 14 13 15	12 ·· 13 15 14
b	13 · 14 15 12	13 ·· 12 14 15	13 ··· 15 12 14
c	14 ·· 15 13 12	14 · 13 12 15	14 ··· 12 15 13
d	15 ·· 14 12 13	15 ··· 13 14 12	15 · 12 13 14

TABLE II.

	1.	2.	3.
a	12 ·· 15 13 14	12 ··· 13 14 15	12 · 14 15 13
b	13 ··· 12 15 14	13 · 15 14 12	13 ·· 14 12 15
c	14 · 12 13 15	14 ·· 13 15 12	14 ··· 15 12 13
d	15 ·· 12 14 13	15 ··· 14 13 12	15 · 13 12 14

TABLE III.

	1.	2.	3.
a	1 · 4 3 2	1 ··· 3 2 4	1 ·· 2 4 3
b	2 ··· 4 1 3	2 ·· 1 3 4	2 · 3 4 1
c	3 · 2 1 4	3 ··· 1 4 2	3 ·· 4 2 1
d	4 ·· 3 1 2	4 · 1 2 3	4 ··· 2 3 1

TABLE IV.

	1.	2.	3.
a	1 ··· 2 3 4	1 · 3 4 2	1 ·· 4 2 3
b	2 · 4 3 1	2 ·· 3 1 4	2 ··· 1 4 3
c	3 ·· 2 4 1	3 ··· 4 1 2	3 · 1 2 4
d	4 · 2 1 3	4 ·· 1 3 2	4 ··· 3 2 1

No combination in any one of these pairs of Tables can be shifted to any combination in the other of the same pair (while the outside numbers remain in proper order) without lifting out of the box and transposing two of the numbers; but any combination can be shifted into any other in the same table by temporarily placing one number in the outer row and shifting the positions of the other three once, or twice.

Tables I and II are for all cases where 1 heads the column, Table III and IV are for cases when 15 heads the column.

In the first and third methods the box is turned till the first four numbers fall on the top line, or the last four on the lower line.

First Method.

Arrangement of numbers in consecutive lines reaching from left to right or from right to left.

Number one heads the column of outside numbers.

Solutions only possible when 12 and 13 are *diagonally* adjacent. The motion of the head of the column must be past 15–12 in the order named, the 1 stopping on the square diagonally adjacent to that occupied by 12. Only those combinations permit solution when the 12 and 13 are *diagonally* adjacent. After the first row is complete, the 5 with its following series pass on the next line pushing the 15–12 before them, and the 9 and following three numbers pass on the third line pushing the 13–14 before them. When this third row is complete the numbers are in order. If after the outside series is complete the middle numbers occur as in any of the combinations of Table I (which are resolvable into each other by moving one of the blocks temporarily to the vacant space on the outside, rotating the others and then replacing it and if necessary repeating the operation by thus temporarily moving one of the blocks which is in its right place and rotating again till the desired combination is effected) the following solutions are possible:

I. *The middle numbers occur as in some combination of Table I.*

1. The increase of outside numbers is with the motion of the hands of a watch; a 1, and b 1 (turn box half round).
2. The increase of outsiders is opposite to the above; c 2, and d 3.

II. *The middle numbers occur in some combination of Table II.*

1. Increase of outside numbers with motion of watch hands c 1, and d 3.
2. Increase of outside numbers opposite to motion of watch hands a 3 and b 2.

Number fifteen heads the column of outside numbers.

Solutions are only possible when the 4 and 3 are *diagonally* adjacent. The 15 passes the 1 and 4 in this order and stops at the square diagonally adjacent to the 4. The 11 passes to the next line pushing the 1–4 before it; the 7 to the third line pushing the 3–2, and when this line is filled the numbers are arranged.

III. *The middle numbers are one of the combinations of Table III.*

1. The decrease of outside numbers with motion of watch hands, c 1 and d 2.
2. The decrease of outside numbers against motion of watch hands, b 3 and a 1.

IV. *The middle numbers are some combination of Table IV.*

1. Decrease of outsiders with motion of watch hands, a 2 and b 1.
2. Decrease of outsiders against motion of watch hands, c 3 and d 1.

Second Method.

Number one heads the outside column.

Solutions only possible when 12 and 14 of the middle numbers are *diagonally* adjacent. Bring the 11 alongside of the 12.

I. *The middle numbers occur as some combination of Table I.*

1. The increase of outside numbers with motion of watch hands; a 3, and c 1.
2. Increase of outside numbers against motion of watch hands; d 1, and b 2.

II. *The middle numbers some combination of Table II.*

1. Increase of outside numbers with motion of watch hands; b 3, and d 1.
2. Increase of outside numbers against motion of watch hands; c 2, and a 1.

Number fifteen leads. Solutions only possible when 1 and 3 are *diagonally* adjacent.

Bring the 5 alongside the 4.

III. *The middle numbers occur as some combination of Table III.*

1. Decrease of outside numbers with motion of watch hands; b 2, and d 1.
2. Decrease of outside numbers against motion of watch hands; a 3, and c 3.

IV. *Middle numbers occur as some combination of Table IV.*

1. Decrease of outside numbers with motion of watch hands; a 3, and c 1.
2. Decrease of outside numbers against motion of watch hands; b 2, and d 2.

Third Method.

The numbers read from left to right on the first and third lines and from right to left on the second and fourth, or *vice versa*, thus:

1	2	3	4
8	7	6	5
9	10	11	12
	15	14	13

The order of outside numbers is 1, 2, 3, 4, 8, 7, 6, 5, 9, 10, 11.

Solutions are only possible when 12 and 15 or 1 and 4 are diagonally adjacent. The first four numbers pass 13–12 in I and II, and 3–4 in III and IV, in the order named, the head of the column coming to rest at the square diagonally adjacent to the 12 or the 4 respectively.

Number one heads the column. The upper line is full.

I. *The middle numbers a combination of Table II.*

1. First four numbers increase with motion of watch hands; a 2, and d 2.
2. First four numbers increase against motion of watch hands; b 1, and c 3.

II. *Middle numbers a combination of Table I.*

1. First four numbers increase with motion of watch hands ; b 3, and c 3.
2. First four numbers increase against motion of watch hands ; a 2, and d 2.

Number fifteen heads the column. Order of outsiders, 15, 14, 13, 12, 8, 9, 10, 11, 7, 6, 5.

III. *Middle numbers a combination of Table IV.*

1. Last four numbers decrease with motion of watch hands ; a 1, and d 3.
2. Last four numbers decrease against motion of watch hands ; b 3, and c 2.

IV. *Middle numbers a combination of Table III.*

1. Last four numbers decrease with motion of watch hands ; b 1, and c 2.
2. Last four numbers decrease against motion of watch hands ; d 3, and a 2.

It is thus seen that there are four tables, each containing twelve combinations of the middle numbers or 48 combinations in all. Each of the three methods of solution takes four combinations from each table or one from every horizontal line, and no combination will permit of but one solution. Since these are all the possible combinations and a solution is given for every one it follows that no other solutions are possible than those above given.

It is but just to say that the first demonstration of the possible solutions of the first method was printed by me in the *Bulletin* of Feb. 26 ; showing that in the 13, 15, 14 difficulty position, two solutions were possible, but that the box must be turned if the 1 was to occupy the left hand upper square. Afterwards a paraphrase of this was printed in the New York *Herald* of Feb. 28, without credit.

Erratum on page 258, 3d line from bottom. For 1000 meters read 1000 feet.

R. Rathbun.